**Analytical queuing models**

**Notation**

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**Variability**

If there were no variability, queues would not have to occur since the capacity of a process could be relatively easily adjusted to match demand

If arrival rate ≤ processing rate && no variation then **WIPq = 0** and **u = 1**

**Utilization = processing rate / (arrival rate · m), m = number of servers**

**Incorporating variability**

Assumption of no variation in arrival or processing times is not realistic. The average or mean arrival and process times can be calculated but only if the variation around these is taken into account – done by using a probability distribution

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The usual measure for indicating the spread of a distribution is its standard deviation σ. To normalize standard deviation, it is divided by the mean of its distribution

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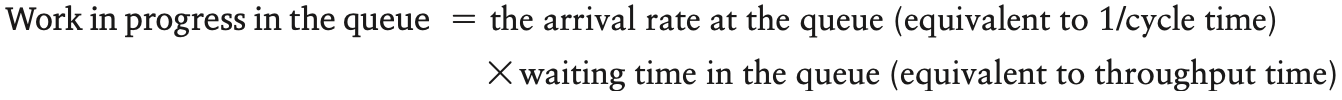
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**Incorporating Little’s law**

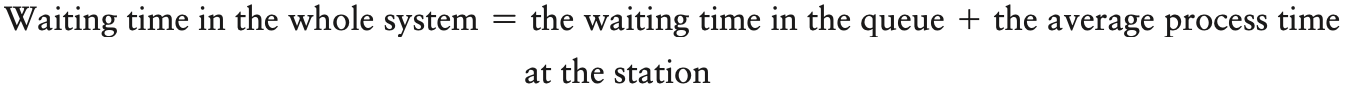
Little’s law: 

Work in progress = Throughput time / Cycle time

WIP = T/C



WIPq = ra x Wq



W = Wq + te

**Types of queueing system**

Queuing systems are characterized by four parameters: A/B/m/b

A = distribution of arrival times (interarrival times, the elapsed times between arrivals)

B = distribution of process time

m = number of servers at each station

b = maximum number of items or people allowed in the system

A or B are usually describe as the:

1. The exponential or Markovian distribution denoted by M
2. The general normal distribution denoted by G

**Kendall’s notation** = M/G/1/5 queuing system indicates a system with exponentially distributed arrivals, process times described as a general distribution such as normal distribution, with one server and a maximum number of items allowed of 5.

The most common situations are:

1. M/M/m = the exponential arrival and processing times with m servers and no maximum limit to the queue
2. G/G/m = general arrival and processing distributions with m servers and no limit to the queue

**M/M/1 queuing systems**

The formula for M/M/1 systems are: En bild som visar Teckensnitt, vit, text, typografi

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Since queue = total throughput time – average processing time, then:

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**M/M/m queuing systems**

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**G/G/1 systems**

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The formula is known as the VUT formule because it describes the waiting time as a function of V = variability in the queuing system, U = utilization of the queuing system (demand vs capacity), and T = processing times at the station

**G/G/m systems**

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